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Structure of the σ meson and the softening

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We study the structure of the σ meson, the lowest-lying resonance of the $\pi\pi$ scattering in the scalar-isoscalar channel, through the softening phenomena associated with the partial restoration of chiral symmetry. We build dynamical chiral models to describe the $\pi\pi$ scattering amplitude, in which the σ meson is described either as the chiral partner of the pion or as the dynamically generated resonance through the $\pi\pi$ attraction. It is shown that the internal structure is reflected in the softening phenomena; the softening pattern of the dynamically generated σ meson is qualitatively different from the behavior of the chiral partner of the pion. On the other hand, in the symmetry restoration limit, the dynamically generated σ meson behaves similarly to the chiral partner.

Keywords: σ meson, chiral dynamics, softening, hadronic molecule

1. Introduction

The study of the structure of hadron resonances is one of the central issues in modern hadron spectroscopy. Among others, the structure of the σ meson has been intensively studied, since the σ meson is considered to play an important role in various aspects of hadron and nuclear physics. For instance, since the scalar-isoscalar excitation of QCD vacuum can be regarded as the amplitude fluctuation of the chiral order parameter $\langle \bar{q}q \rangle$, the nature of the σ meson is crucial to understand the dynamical chiral

2

symmetry breaking in QCD.[1] While the existence of the scalar-isoscalar resonance in the $\pi\pi$ scattering is established by recent analyses (see e.g. Ref. 2), its internal structure is still controversial. Thus, it is our aim here to investigate the structure of the σ meson.

There are many proposals for the structure of the σ meson based on various effective models: collective $\bar{q}q$ excitation,[1,3,4] tetraquark structure with strong diquark correlation,[5] dynamically generated $\pi\pi$ resonance,[6] and so on. Conventionally, the validity of certain structure has been tested by comparing the model prediction with experimental data, for instance, mass spectrum and the decay properties.

It is also possible to investigate the structure of the resonance from the response to the change of the internal/external parameter of the models. For instance, the study of the N_c scaling is successful to disentangle the $\bar{q}q$ and other structures for meson resonances.[7] Following this philosophy, we would like to study the spectral change of the σ meson when the chiral symmetry is partially restored, in order to discriminate the different internal structures.

In association with chiral symmetry restoration, the softening of the σ meson has been discussed.[1,4,8] In the linear realization of chiral symmetry, the σ meson forms a chiral four-vector together with the pion, and hence they are chiral partners. The partial restoration of chiral symmetry induces the softening of the σ spectrum, which results in the enhancement of the $\pi\pi$ cross section in the scalar-isoscalar channel near threshold. It was shown later that the threshold enhancement in the $I = J = 0$ channel takes place also in the nonlinear realization of chiral symmetry without the bare σ field[9,10] where the σ meson is expressed as a dynamically generated resonance from the attractive $\pi\pi$ interaction. Although similar threshold enhancement of the cross section is observed in both cases, the mechanism which causes the softening is quite different. Based on these observations, we demonstrate that the softening phenomena reflects the structure of the σ meson, paying attention to the nature of the s-wave resonance.

When the symmetry is completely restored, the chiral partners emerge as the pair of particles with a degenerate mass. If we regard the mass degeneracy as the condition for the chiral partner, we can extend the notion of the chiral partner for the dynamically generated σ meson in the nonlinear realization. We will study the structure of the $\pi\pi$ scattering amplitude of our model in the restoration limit, to discuss the chiral partner of the pion.

2. Formulation

Here we describe the $\pi\pi$ scattering amplitude with the σ resonance in the $I = J = 0$ channel. We consider the low energy behavior of the amplitude based on chiral effective Lagrangian, and then introduce the unitarity condition to extend the applicability of the model to the resonance energy region.

We start from the Lagrangian of two-flavor linear sigma model to derive the $\pi\pi$ scattering amplitudes:

$$\mathcal{L} = \frac{1}{4}\text{Tr}\left[\partial M \partial M^\dagger - \mu^2 M M^\dagger - \frac{2\lambda}{4!}(M M^\dagger)^2 + h(M + M^\dagger)\right], \quad (1)$$

where $M = \sigma + i\boldsymbol{\tau} \cdot \boldsymbol{\pi}$. In this Lagrangian, the σ meson is treated as the chiral partner of the pion. For negative μ^2, chiral symmetry is spontaneously broken and three parameters in the Lagrangian μ, λ, and h are related to the chiral condensate (pion decay constant) $\langle\sigma\rangle = f_\pi$, the mass of the pion m_π, and the mass of the σ meson m_σ in the mean-field level.

Crossing symmetry enables us to express the general $\pi\pi$ scattering amplitude as $T_{\text{tree}}(s,t,u) = A(s,t,u)\delta_{ab}\delta_{cd} + A(t,s,u)\delta_{ac}\delta_{bd} + A(u,t,s)\delta_{ad}\delta_{bc}$, where the invariant amplitude A is given, from the Lagrangian (1) at tree level, by[9]

$$A(s) = \frac{s - m_\pi^2}{\langle\sigma\rangle^2} - \frac{(s - m_\pi^2)^2}{\langle\sigma\rangle^2}\frac{1}{s - m_\sigma^2}.$$

In this expression, the first (second) term can be regarded as the leading (higher) order contribution in the chiral perturbation theory. The coefficient of the leading order term is fixed by the low energy theorem, while the low energy constant for the higher order terms is not constrained by the symmetry and should be determined by experiments. Thus, we introduce a parameter x to express the general amplitude

$$A(s;x) = \frac{s - m_\pi^2}{\langle\sigma\rangle^2} - x\frac{(s - m_\pi^2)^2}{\langle\sigma\rangle^2}\frac{1}{s - m_\sigma^2}. \quad (2)$$

By choosing $x = 1$, we recover the result of the original Lagrangian (1). If we take $x = 0$, then we are left with the leading order interaction, which also corresponds to the heavy m_σ limit. In this way, we can smoothly connect the original linear sigma model ($x = 1$) and the leading order chiral perturbation theory without the bare σ field ($x = 0$). Projecting Eq. (2) onto the $I = J = 0$ channel, we obtain the tree-level amplitude for the $\pi\pi$

4

scattering as

$$T_{\text{tree}}(s;x) = \frac{m_\sigma^2 - m_\pi^2}{\langle\sigma\rangle^2}\left[\frac{2s - m_\pi^2}{m_\sigma^2 - m_\pi^2}(1-x) - 5x\right.$$
$$\left. - 3x\frac{m_\sigma^2 - m_\pi^2}{s - m_\sigma^2} - 2x\frac{m_\sigma^2 - m_\pi^2}{s - 4m_\pi^2}\ln\left(\frac{m_\sigma^2}{m_\sigma^2 + s - 4m_\pi^2}\right)\right], \quad (3)$$

in the center-of-mass frame.

Next we consider the unitarity condition $\text{Im }T^{-1}(s) = -\Theta(s)/2$ for $s > 4m_\pi^2$, with the two-body phase space function $\Theta(s) = (16\pi)^{-1}\sqrt{1 - 4m_\pi^2/s}$. Based on the N/D method, we write down the general expression of the unitary scattering amplitude $T(s;x)$. Matching the chiral interaction $T_{\text{tree}}(s;x)$ with the loop expansion of the full amplitude $T(s;x)$, we obtain the amplitude which is consistent with both chiral low energy theorem and unitarity as[11]

$$T(s;x) = \frac{1}{T_{\text{tree}}^{-1}(s;x) + G(s)},$$
$$G(s) = \frac{1}{2}\frac{1}{(4\pi)^2}\left\{a(\mu) + \ln\frac{m_\pi^2}{\mu^2} + \sqrt{1 - \frac{4m_\pi^2}{s}}\left[\ln\frac{\sqrt{1 - \frac{4m_\pi^2}{s}} + 1}{\sqrt{1 - \frac{4m_\pi^2}{s}} - 1}\right]\right\},$$

where $a(\mu)$ is the subtraction constant at the subtraction point μ. We determine the subtraction constant by excluding the nontrivial CDD pole (states which does not originate in the two-body dynamics) in the amplitude[12]

$$G(s) = 0 \quad \text{at} \quad s = m_\pi^2, \quad (4)$$

which leads to $a(m_\pi) = -\pi/\sqrt{3}$. With this subtraction constant, the full scattering amplitude T reduces into the tree level one T_{tree} at $s = m_\pi^2$.

The full amplitude $T(s;x)$ corresponds to the nonperturbative resummation of the s-channel loop diagrams up to infinite order. For the $x = 1$ case, the bare σ pole in the Lagrangian acquire the finite width through the coupling to the $\pi\pi$ state, and the full scattering amplitude exhibits a pole in the complex energy plane. On the other hand, for the $x = 0$ case without the bare σ pole, a resonance can be dynamically generated as the pole of the amplitude, if the two-body interaction is sufficiently attractive. Indeed, it is shown that the resummation of the leading order interaction generates the σ meson dynamically.[13] In the following we compare the properties of these σ states: one originating in the chiral partner of the pion in the linear σ model ($x = 1$), and another generated dynamically from the attractive $\pi\pi$ interaction ($x = 0$).

3. Chiral symmetry restoration

3.1. *Prescription for the symmetry restoration*

Now we introduce the effect of chiral symmetry restoration through the modification of the model parameters. It is known that the chiral condensate should decrease with the chiral symmetry restoration, so we parametrize the condensate by

$$\langle\sigma\rangle = \Phi\langle\sigma\rangle_0,$$

where $\langle\sigma\rangle_0$ is the condensate in vacuum and we vary the parameter Φ from one to zero to express the symmetry restoration. The NJL model indicates that m_π hardly changes as symmetry restoration,[1] so we assume that it is a constant:

$$m_\pi = \text{const.}$$

The bare mass of the σ should be degenerated with pion when the symmetry is restored. This can be achieved by the mean-field relation of Eq. (1)

$$m_\sigma = \sqrt{\lambda\frac{\langle\sigma\rangle^2}{3} + m_\pi^2},$$

with λ and m_π being fixed.

3.2. *Behavior of the amplitude in the restoration limit*

It is instructive to study the behavior of the $\pi\pi$ scattering amplitude in the limit $\langle\sigma\rangle \to 0$. Here we analytically derive the pole of the amplitude in the restoration limit. In subsection 4.2, we will numerically demonstrate that the obtained result corresponds to the asymptotic behavior of the σ pole for $\langle\sigma\rangle \to 0$.

We first consider the $x = 1$ case, where the σ meson is the chiral partner of the pion. To make the $\langle\sigma\rangle$ dependence in m_σ explicit, we rewrite the tree level amplitude as

$$T_{\text{tree}}(s;1) = -\frac{5\lambda}{3} - \frac{\lambda^2\langle\sigma\rangle^2}{3}\frac{1}{s - m_\pi^2 - \frac{\lambda}{3}\langle\sigma\rangle^2}$$
$$- \frac{2\lambda^2\langle\sigma\rangle^2}{9}\frac{1}{s - 4m_\pi^2}\ln\frac{m_\pi^2 + \frac{\lambda}{3}\langle\sigma\rangle^2}{s - 3m_\pi^2 + \frac{\lambda}{3}\langle\sigma\rangle^2}.$$

The second term represents the bare pole of the σ meson. As $\langle\sigma\rangle \to 0$, the pole mass of this term decreases and finally it coincides with the pion mass. Because of the renormalization condition (4), the tree-level amplitude $T_{\text{tree}}(s;1)$ coincides with the full amplitude $T(s;1)$ at $s = m_\pi^2$, so the

6

full amplitude $T(s; 1)$ also has a pole as in the same way with $T_{\text{tree}}(s; 1)$. Approximating the amplitude by the Breit-Wigner form around the pole,

$$T(s; 1) \sim -\frac{g^2}{s - M_{\text{pole}}^2},$$

we extract the mass of the state M_{pole} and the coupling to the scattering state g. In the present case, we find

$$g \to 0, \quad M_{\text{pole}} \to m_\pi \quad \text{for} \quad \langle\sigma\rangle \to 0.$$

This is what we anticipate for the properties of the chiral partner; the mass degeneracy with the pion and the vanishing of the coupling constant to the $\pi\pi$ state.

Next we consider the $x = 0$ case without the bare σ pole. In this case, $\langle\sigma\rangle$ dependence of the tree-level amplitude (3) exclusively stems from the overall factor,

$$T_{\text{tree}}(s; x) \propto \frac{1}{\langle\sigma\rangle}.$$

Taking the restoration limit $\langle\sigma\rangle \to 0$, this term diverges, and therefore the full amplitude is solely determined by the loop function $G(s)$:

$$T(s; x) = \frac{1}{T_{\text{tree}}^{-1}(s, x) + G(s)} \to \frac{1}{G(s)} \quad \text{for} \quad \langle\sigma\rangle \to 0.$$

Thus, the pole of the amplitude in the restoration limit is given by the zero of $G(s)$. The present renormalization scheme requires $G(s) = 0$ for $s = m_\pi^2$, which indicates the existence of a pole at $\sqrt{s} = m_\pi$ in the σ channel. The coupling constant g can be obtained by calculating the residue of the pole:

$$g^2|_{\langle\sigma\rangle \to 0} = (4\pi)^2 \left(\frac{\pi}{3\sqrt{3}} - \frac{1}{2}\right)^{-1} m_\pi^2. \tag{5}$$

Thus, for the dynamically generated σ meson, the amplitude has a pole at the pion mass with the coupling constant which is proportional to m_π for $\langle\sigma\rangle \to 0$.

This result has an interesting implication for the chiral partner. Strictly speaking, the notion of the "chiral partner" is defined only in the chiral limit $(m_\pi \to 0)$, where the SU(2)×SU(2) symmetry is exact in the Wigner phase. In the chiral limit, Eq. (5) indicates $g^2|_{\langle\sigma\rangle \to 0} = 0$, so the asymptotic value of the mass and coupling constant of the dynamically generated σ meson is exactly the same with the chiral partner case. Namely, the dynamically generated σ meson behaves as if it is the chiral partner of the pion, for $m_\pi \to 0$.

4. Numerical study for the softening phenomena

4.1. *Structure of the σ meson in vacuum*

We first show the description of the scattering amplitude in vacuum. We choose canonical values of the parameters as $\langle \sigma \rangle_0 = 93$ MeV, $m_\pi = 140$ MeV, and $m_\sigma = 550$ MeV. By taking the parameter $x = 1$, the σ meson is described as the chiral partner of the pion (chiral σ), and we refer to this case as "model A". Choosing $x = 0$, we obtain the dynamically generated σ meson in the amplitude. This case is called as "model B".

To check the agreement with the physical amplitude, we study the properties of these models without symmetry restoration. We calculate the scattering length $a = \frac{1}{32\pi} T(4m_\pi^2)$ (in units of m_π^{-1}) in these models. The results are shown in Table 1, together with the pole position of the amplitude in the complex energy plane. We find a qualitative agreement with the recent analyses of experimental data,[2,14] $a_{\exp} \sim 0.216$ and $z = 441 - 272i$ MeV.

4.2. *Softening of the σ meson*

We then study the variation of the scattering amplitude in the σ channel along with partial restoration of chiral symmetry. We plot the reduced cross section $\bar{\sigma} = |T|^2/s$ and the trace of the pole position as functions of the total center-of-mass energy $\sqrt{s}$, by changing the parameter Φ from 1 to 0.

The invariant mass spectra and the pole trajectory of model A are shown in Fig. 1, where the softening of σ is clearly observed. As the symmetry is restored, the σ pole moves toward the $\pi\pi$ threshold and the spectrum gets narrow and enhanced around threshold. In this case, since the σ meson is treated as the chiral partner of pion, the softening phenomena is driven by the decrease of the bare mass of the σ and its consequence of the reduction of the phase space for the decay.[8] In the limit $\langle \sigma \rangle \to 0$, the pole approaches the mass of the pion, as indicated by the analysis in section 3.2.

We show the results of model B in Fig. 2. In this case, although the threshold enhancement takes place, the change of the spectrum as well as the trace of the pole are qualitatively different from those of model A.

Table 1.   Properties of the models: value of parameter x, possible origin of the pole, the scattering length a in units of m_π^{-1}, and the pole position of the amplitude in vacuum.

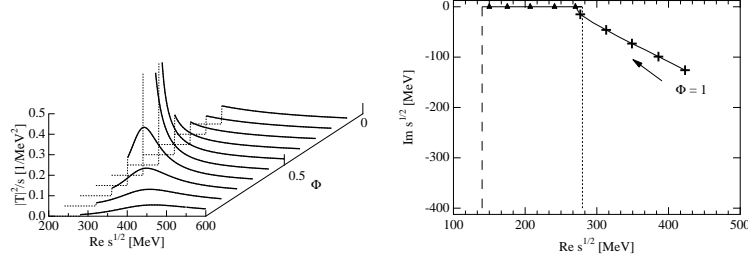|	x	origin	$a\ [m_\pi^{-1}]$	pole position [MeV]
model A	1	chiral partner	0.244	$423 - 126i$
model B	0	dynamically generated	0.174	$364 - 356i$

8

Fig. 1.   Mass spectra of the σ meson (left) and the trace of the pole positions (right) in model A ($x = 1$). The poles on the first (second) Riemann sheet are plotted by triangles (crosses). Symbols are marked with each 0.1 step of Φ. Arrow indicates the direction of the movement of the pole as the parameter Φ is decreased from 1 to 0. Dotted (dashed) line represents the energy corresponds to the threshold (mass of pion).
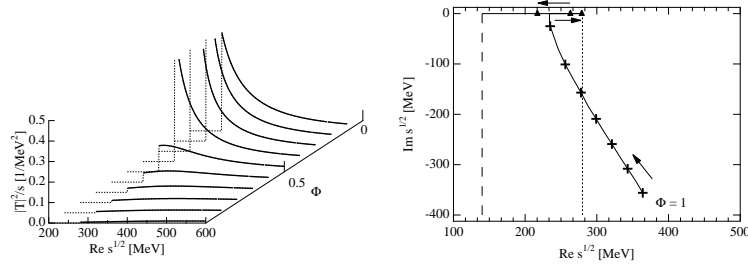

Fig. 2.   Mass spectra of the σ meson (left) and the trace of the pole positions (right) in model B ($x = 0$). The poles on the first (second) Riemann sheet are plotted by triangles (crosses). Symbols are marked with each 0.1 step of Φ. Arrows indicate the direction of the movement of the pole as the parameter Φ is decreased from 1 to 0. Dotted (dashed) line represents the energy corresponds to the threshold (mass of pion).

Especially, the pole in the complex energy plane goes to the subthreshold energy region, keeping the width finite. This is a peculiar feature of the dynamically generated σ meson. As a consequence, the strong enhancement of the cross section occurs at much later stage of the symmetry restoration, compared with the model A.

Let us discuss how this structure appears in model B. The mechanism of the threshold enhancement in the nonlinear σ model has been studied in Ref. 9; the partial restoration of chiral symmetry induces the reduction of the pion decay constant. Since the low energy interaction is proportional to $1/\langle\sigma\rangle^2$, the symmetry restoration effectively increases the attractive interaction. Thus, the dynamically generated σ resonance will eventually turns into a $\pi\pi$ bound state when the interaction becomes sufficiently attractive.

At this stage, however, it is important to recall that the σ is in s-wave. In this case, when the attraction is increased, the resonance first becomes the *virtual state* which is the pole on the second Riemann sheet of the energy plane but lies below the threshold.[15] The peculiar pole trajectory in Fig. 2 is due to the appearance of the virtual state.

It is worth mentioning that the finite pion mass is important for the appearance of the virtual state. If $m_\pi = 0$, the $\pi\pi$ threshold lies at $\sqrt{s} = 0$, so there is no region where the virtual state appears. Indeed, the virtual state was not seen in the analysis of the dynamically generated σ meson in Ref. 10, studied in the chiral limit. In addition, the s-wave nature of the σ is also essential for the virtual state. Therefore, the behavior of the ρ meson in p-wave amplitude will not exhibit such a structure (see also Ref. 16).

As we further restore the symmetry, the σ meson becomes the bound state. In the limit $\langle \sigma \rangle \to 0$, the pole moves toward the pion mass. This is again in agreement with the result in section 3.2: the appearance of the pole at the pion mass in the restoration limit.

5. Summary

We have studied the properties of the σ meson in the $\pi\pi$ scattering associated with the restoration of chiral symmetry. Two models are constructed based on chiral low energy interaction and unitarity of the scattering amplitude: one describes the σ meson as the chiral partner of the pion, and the other treats the σ meson as dynamically generated resonance.

For the dynamically generated σ meson, we find the qualitative difference from the chiral partner σ in the softening behavior, namely, the movement of the pole of the amplitude and its consequence of the change of the spectrum. The difference stems from the mechanism which drives the softening, and it is the appearance of the virtual state that leads to the distinct behavior of the dynamically generated σ.

We also study the asymptotic properties of the σ pole in the restoration limit. For the σ meson as the chiral partner, we find that the mass of the σ pole approaches the pion mass and the coupling to the $\pi\pi$ state vanishes. For the dynamically generated σ, we also find the mass degeneracy with the pion, and the coupling strength vanishes in the chiral limit $m_\pi \to 0$. Namely, the behavior of the dynamically generated σ pole in the restoration limit is essentially the same with what we expect for the chiral partner. This is a nontrivial result which urge us to speculate the possibility of the dynamically generated σ meson as the chiral partner of the pion.

In this way, through the comparison of two models, we draw two con-

10

clusions: (i) with partial restoration of chiral symmetry, the difference of the internal structures is reflected in the spectral change of the σ channel, and (ii) in the symmetry restoration limit, the difference of the structure is reduced and we obtain essentially the same behavior of the σ pole for $m_\pi \to 0$. More comprehensive analysis, including the case with the σ meson as the CDD pole, is now underway.[17]

Acknowledgments

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